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THE MONIST

THE RELATIVITY OF SPACE.[1]

I.

IT is impossible to represent to oneself empty space; all our efforts to imagine a pure space, from which the changing images of material objects would be excluded, can result only in a representation where vividly colored surfaces, for example, are replaced by lines of faint coloration, and we cannot go to the very end in this way without all vanishing and terminating in nothingness.

Thence comes the irreducible relativity of space.

Whoever speaks of absolute space uses a meaningless phrase. This is a truth long proclaimed by all who have reflected upon the matter, but which we are too often led to forget.

I am at a definite point in Paris, the Place du Panthéon for instance, and I say, I shall come back *here* to-morrow." If I am asked, "Do you mean you will return to the same point of space?" I shall be tempted to answer, "Yes"; and yet I shall be wrong, since by to-morrow the earth will have journeyed hence carrying with it the Place du Panthéon, which will have traveled over more than 2 million kilometers. And if I tried to speak more precisely I should gain nothing, since our globe has run over these 2 million kilometers in its motion with relation to the sun, while the sun in its turn is displaced with reference to the Milky Way, while the Milky Way itself is doubtless in motion

[1] Translated by George Bruce Halsted.

without our being able to perceive its velocity. So that we are completely ignorant, and always shall be, of how much the Place du Panthéon is displaced in a day.

In sum, I meant to say: "To-morrow I shall see again the dome and the pediment of the Panthéon, and if there were no Panthéon my phrase would be meaningless and space would vanish."

This is one of the most commonplace forms of the principle of the relativity of space; but there is another, upon which Delbœuf has particularly insisted. Suppose that in the night all the dimensions of the universe become a thousand times greater; the world will have remained *similar* to itself, giving to the word *similitude* the same meaning as in Euclid, Book VI. Only what was a meter long will measure thenceforth a kilometer, what was a millimeter long will become a meter. The bed whereon I lie and my body itself will be enlarged in the same proportion. When I awake to-morrow morning, what sensation shall I feel in the presence of such an astounding transformation? I shall perceive nothing at all. The most precise measurements will be incapable of revealing to me anything of this immense convulsion, since the measures I use will have varied precisely in the same proportion as the objects I seek to measure. In reality, this convulsion exists only for those who reason as if space were absolute. If I for a moment have reasoned as they do, it is in order the better to bring out that their way of seeing implies contradiction. In fact it would be better to say that space being relative, nothing at all has happened, which is why we have perceived nothing.

Have we the right, therefore, to say that we know the distance between two points? No, since this distance could undergo enormous variations without our being able to perceive them, provided the other distances have varied in the same proportion. We have just seen that when I say,

"I shall be here to-morrow," this does not mean that to-morrow I shall be at the same point of space where I am to-day, but rather that to-morrow I shall be at the same distance from the Panthéon as to-day. And we see that this statement is no longer sufficient and that I should say: "To-morrow and to-day my distance from the Panthéon will be equal to the same number of times the height of my body."

But this is not all. I have supposed the dimensions of the world to vary, but that at least the world would always remains similar to itself. We might go much further, and one of the most astonishing theories of modern physics furnishes us the occasion.

According to Lorentz and Fitzgerald, all the bodies borne along in the motion of the earth undergo a deformation. This deformation is, in reality, very slight, since all dimensions parallel to the movement of the earth diminish by a hundred millionth, while the dimensions perpendicular to this movement are unchanged.

But it matters little that it is slight; that it exists suffices for the conclusion I am about to draw. Moreover, I have said it was slight, but in reality I know nothing about it; I have myself been victim of the tenacious illusion which makes us believe that we can conceive an absolute space; I have thought of the motion of the earth in its elliptic orbit around the sun, and I have allowed thirty kilometers as its velocity. But its real velocity (I mean, this time, not its absolute velocity, which is meaningless, but its velocity with relation to the ether) I do not know, and have no means of knowing it: It may be 10 or 100 times greater, and then the deformation will be 100 or 10,000 times more.

Can we show this deformation? Evidently not. Here is a cube with its edges 1 meter in length; in consequence of the earth's displacement it is deformed, the edge which is parallel to the motion becoming smaller, the others re-

maining unchanged. If I wish to assure myself of this by the aid of a meter measure, I shall first measure one of the edges perpendicular to the motion and shall find that my standard meter fits this edge exactly; and in fact neither of these two lengths is changed, since both are perpendicular to the motion. Then I wish to measure the other edge, that parallel to the motion; to do this I change the position of my meter and turn it so as to apply it to the edge. But since the meter has changed orientation and become parallel to the motion, it has undergone, in its turn, the deformation, so that though the edge be not exactly a meter long, it will fit exactly and I shall find out nothing.

You ask then of what use is the hypothesis of Lorentz and Fitzgerald if no experiment can make its verification possible? It is my exposition that has been incomplete. I have spoken only of measurements that can be made with a meter; but we can also measure a length by the time it takes light to traverse it, on condition that we suppose the velocity of light constant and independent of direction.

Lorentz could have accounted for the facts by supposing the velocity of light greater in the direction of the earth's motion than in the perpendicular direction. He preferred to suppose that the velocity is the same in these different directions, but that the bodies are smaller in the one than in the other.

If the wave surfaces of light had undergone the same deformations as the material bodies we should never have perceived the Lorentz-Fitzgerald deformation.

In either case it is not a question of absolute magnitude but of the measure of this magnitude by means of some instrument which may be a meter or the path traversed by light. It is only the relation of the magnitude to the instrument that we measure; and if this relation is altered, we have no way of knowing whether it is the magnitude

or the instrument which has changed. But what I wish to bring out is that in this deformation the world has not remained similar to itself; squares have become rectangles, circles ellipses, spheres ellipsoids. And yet we have no way of knowing whether this deformation is a real one. Evidently we might go much further. In place of the Lorentz-Fitzgerald deformation whose laws are particularly simple, we might imagine any deformation whatsoever. Bodies might be deformed according to any laws however complicated and we should never notice it provided all bodies without exception were deformed according to the same laws. In saying "all bodies without exception" I include of course our own body and the light rays emanating from different objects.

If we look at the world in one of those mirrors of complicated shape which deform objects in a bizarre way, the mutual relations of the different parts of this world would not be altered; if in fact two real objects touch, their images likewise seem to touch. Of course when we look in such a mirror we see indeed the deformation, but this is because the real world continues to exist alongside of its deformed image. Then, too, even if this real world were hidden from us, there is one thing that could not be hidden, and that is ourself; we could not cease to see, or at least to feel, our body and our limbs which have not been deformed and which continue to serve us as instruments of measure. But if we imagine our body itelf deformed in the same way as if seen in the mirror, these instruments of measure in their turn will fail us and the deformation will no longer be ascertainable.

Consider in the same way two worlds as images of one another. Each object P of the world A has a corresponding image P′ in the world B. The coordinates of this image P′ are determinate functions of those of the object P. Moreover they may be any functions whatsoever; I only sup-

pose them chosen once for all. Between the position of P and that of P′ there is a constant relation. What this relation is does not matter; enough that it be constant.

These two worlds will be indistinguishable one from the other; in other words, the first will be for its inhabitants what the second is for its. This will continue to be the case as long as the two worlds remain strangers to each other.

Suppose we live in a world A and have constructed our science and in particular our geometry. In the meantime the inhabitants of world B will have constructed a science, and as their world is the image of ours their geometry will also be the image of ours or, better, it will be the same as ours. But if some day a window is opened for us upon world B, how we shall pity them! "Poor things," we shall say, "they think they have made a geometry, but what they call so is only a grotesque image of our own; their straights are all twisted, their circles are humped, their spheres have capricious inequalities." And we shall never suspect that they say the same of us, and no one will ever know who is right.

We see in how broad a sense the relativity of space should be understood. Space is in reality amorphous and only the things which are in it give it form. What then should be thought of that direct intuition we should have of the straight or of distance? So little do we have any intuition of distance in itself that in the night, as we have said, a distance might become a thousand times greater without our being able to perceive it, if all other distances had undergone the same alteration. And in a night the world B might even be substituted for the world A without our having any way of knowing it, and then the straight lines of yesterday would have ceased to be straight and we should never be the wiser.

One part of space is not, by itself and in the absolute

sense of the word, equal to another part of space; because if it is so for us, it would not be so for the dwellers in the world B; and these have just as much right to reject our opinion as we have to condemn theirs.

I have elsewhere[2] shown what are the consequences of these facts from the view-point of non-Euclidean geometry and other analogous geometries; to that I do not care to return; and to-day I shall take a somewhat different point of view.

II.

If this intuition of distance, of direction, of the straight line—if this direct intuition of space, in a word—does not exist, whence comes our belief that we have it?

If this is only an illusion, why is this illusion so tenacious? It is proper to examine into this. We have said there is no direct intuition of size and we can arrive at only the relation of this magnitude to our instruments of measure. We should therefore not have been able to construct space if we had not had an instrument to measure it; and this instrument to which we relate everything, which we use instinctively, is our own body.

It is in relation to our body that we place exterior objects, and the only spatial relations of these objects that we can represent are their relations to our body. It is our body which serves us, so to speak, as system of axes of coordinates.

For example at one instant α, the presence of the object A is revealed to me by the sense of sight; at another instant β, the presence of another object B is revealed to me by another sense, that of hearing or of touch, for instance. I judge that this object B occupies the same place as the object A. What does this mean? First it does not signify that these two objects occupy, at two different

[2] In *Science et hypothèse* and *La valeur de la science.*—Tr.

moments, the same point of an absolute space, which even if it existed would escape our cognition, since between the instants α and β the solar system has moved and we cannot know its displacement. It means that these two objects occupy the same relative position with reference to our own body.

But even this, what does it mean? The impressions that have come to us from these objects have followed absolutely different paths, the optic nerve for the object A, the acoustic nerve for the object B. They have nothing in common from the quantitative point of view. The representations we are able to make of these two objects are absolutely heterogeneous, irreducible one to the other. Only I know that to reach the object A, I have merely to extend the right arm in a certain way. Even when I abstain from doing it, I represent to myself the muscular and other analogous sensations which would accompany this extension, and this representation is associated with that of the object A.

Now I likewise know I can reach the object B by extending my right arm in the same manner, an extension accompanied by the same train of muscular sensations. And when I say these two objects occupy the same place, I mean nothing more.

I also know I could have reached the object A by another appropriate motion of the left arm, and I represent to myself the muscular sensations which would have accompanied this movement; and by this same motion of the left arm accompanied by the same sensations, I likewise could have reached the object B.

And that is very important, since thus I can defend myself against dangers menacing me from the object A or the object B. With each of the blows that can hit us, nature has associated one or more parries which permit of our guarding ourselves. The same parry may respond

to several strokes; and so it is, for instance, that the same motion of the right arm would have allowed us to guard at the instant α against the object A and at the instant β against the object B. Just so the same stroke can be parried in several ways, and we have said, for instance, the object A could be reached indifferently either by a certain movement of the right arm or by a certain movement of the left arm.

All these parries have nothing in common except warding off the same blow, and this and nothing else is meant when we say they are movements terminating at the same point of space. Likewise, these objects, which we say occupy the same point of space, have nothing in common, except that the same parry guards against them.

Or, if you choose, imagine innumerable telegraph wires, some centripetal, others centrifugal. The centripetal wires warn us of accidents happening without; the centrifugal wires carry the reparation. Connections are so established that when a centripetal wire is traversed by a current this acts on a relay and so starts a current in one of the centrifugal wires, and things are so arranged that several centripetal wires may act on the same centrifugal wire if the same remedy suits several ills, and that a centripetal wire may agitate different centrifugal wires, either simultaneously or in lieu of one another when the same ill may be cured by several remedies.

This complex system of associations, this table of distribution, so to speak, comprises all our geometry, or, if you wish, all in our geometry that is instinctive. What we call our intuition of the straight line or of distance is the consciousness we have of these associations and of their imperious character. It is easy to understand whence comes this imperious character itself. An association will seem to us by so much the more indestructible as it is more ancient. But these associations are not for the most

part conquests of the individual, since their trace is seen in the new-born babe; they are conquests of the race. Natural selection had to bring about these conquests the more quickly, the more necessary they were.

On this account, those of which we speak must have been among the earliest in date, since without them the defence of the organism would have been impossible. From the time when the cellules were no longer merely juxtaposed, but were called upon to give mutual aid, it was needful that a mechanism be organized analogous to that we have described, so that this aid would not miss its way, but forestall the peril.

When a frog is decapitated, and a drop of acid is placed on a point of its skin, it seeks to wipe off the acid with the nearest foot, and if this foot be amputated, it sweeps it off with the foot of the opposite side. Here we have the double parry of which I have just spoken, allowing the combating of an ill by a second remedy, if the first fails. It is this multiplicity of parries, and the resulting coordination, which is space.

We see to what depths of the unconscious we must descend to find the first traces of these spatial associations, since only the inferior parts of the nervous system are involved. Why be astonished then at the resistance we oppose to every attempt to dissociate what so long has been associated? Now, it is just this resistance that we call the evidence for the geometric truths. This evidence is nothing but the repugnance we feel toward breaking with very old habits which have always proved good.

III.

The space so created is only a little space extending no farther than my arm can reach; the intervention of the memory is necessary to push back its limits. There are points which will remain out of my reach, whatever effort I make

to stretch forth my hand; if I were fastened to the ground like a hydropolyp, for instance, which can only extend its tentacles, all these points would be outside of space, since the sensations we could experience from the action of bodies there situated, would be associated with the idea of no movement allowing us to reach them, of no appropriate parry. These sensations would not seem to us to have any spatial character, and we should not seek to localize them.

But we are not fixed to the ground like some of the lower animals; we can, if the enemy be too far away, advance toward him first and extend a hand when we are sufficiently near. This is still a parry, but a parry at long range.

On the other hand, it is a complex parry, and into the representation we make of it enters the representation of the muscular sensations caused by the movements of the legs, that of the muscular sensations caused by the final movement of the arm, that of the sensations of the semicircular canals, etc. We must besides represent to ourselves, not a complex of simultaneous sensations, but a complex of successive sensations, following each other in a determinate order, and this is why I have just said that the intervention of memory was necessary.

Notice moreover that to reach the same point I may approach nearer the mark to be attained, so as to have to stretch my arm less. What more? It is not one, it is a thousand parries I can oppose to the same danger. All these parries are made of sensations which may have nothing in common and yet we regard them as defining the same point of space, since they may respond to the same danger and are all associated with the notion of this danger. The potentiality of warding off the same stroke makes the unity of these different parries, just as the possibility of being parried in the same way makes the unity of such

different kinds of strokes which may menace us from the same point of space. It is this double unity which makes the individuality of each point of space, and, in the notion of point, there is nothing else.

The space before considered, which might be called *restricted space,* was referred to coordinate axes attached to my body; these axes were fixed, since my body did not move and only my members changed their position.

What are the axes to which we naturally refer extended space, that is to say, the new space just defined? We define a point by the sequence of movements to be made to reach it, starting from a certain initial position of the body. The axes are therefore fixed to this initial position of the body.

But the position I call initial may be arbitrarily chosen among all the positions my body has successively occupied; if the more or less unconscious memory of these successive positions is necessary for the genesis of the notion of space, this memory may go back more or less remotely into the past. Thence results in the definition itself of space a certain indeterminateness, and it is precisely this indetermination which constitutes its relativity.

There is no absolute space; there is only space relative to a certain initial position of the body. For a conscious being fixed to the ground like some of the lower animals, and consequently knowing only restricted space, space would still be relative (since it would have reference to his body), but this being would not be conscious of this relativity, because the axes of reference for this restricted space would be unchanging. Doubtless the rock to which this being would be fettered would not be motionless, since it would be carried along in the movement of our planet; for us consequently these axes would change at each instant; but for him they would be changeless.

We have the faculty of referring our extended space

now to the position A of our body, considered as initial, again to the position B, which it had some moments afterward, and which we are free to regard in its turn as initial; we make therefore at each instant unconscious transformations of coordinates. This faculty would be lacking in our imaginary being, and from not having traveled, he would think space absolute. At every instant, his system of axes would be imposed upon him; this system would have to change greatly in reality, but for him it would be always the same, since it would be always the *only* system. Quite otherwise is it with us, who at each instant have many systems among which we may choose at will, on condition of going back by memory more or less far into the past.

This is not all. Restricted space would not be homogeneous; the different points of this space could not be regarded as equivalent, since some could be reached only at the cost of the greatest efforts, while others could be easily attained. On the contrary, our extended space seems to us homogeneous, and we say all points are equivalent.

What does this mean? If we start from a certain place A, we can, from this position, make certain movements M, characterized by a certain complex of muscular sensations. But, starting from another position B, we make movements M′ characterized by the same muscular sensations. Let a, then, be the situation of a certain point of the body, the end of the index finger of the right hand for example, in the initial position A, and b the situation of this same index when, starting from this position A, we have made the motions M. Afterwards, let a' be the situation of this index in the position B, and b' its situation when, starting from the position B, we have made the motions M′.

I am accustomed to say that the points of space a and b are related to each other just as are the points a' and b', and this simply means that the two series of movements

M and M′ are accompanied by the same muscular sensations. And as I am conscious that, in passing from the position A to the position B, my body has remained capable of the same movements, I know there is a point of space related to the point a' just as any point b is to the point a, so that the two points a and a' are equivalent. This is what is called the homogeneity of space. And at the same time this is why space is relative, since its properties remain the same whether it be referred to the axes A or to the axes B, so that the relativity of space and its homogeneity are one sole and same thing.

Now if I wish to pass to the great space, which no longer serves only for me, but where I may lodge the universe, I get there by an act of imagination. I imagine how a giant would feel who could reach the planets in a few steps; or, if you choose, what I myself should feel in presence of a miniature world where these planets were replaced by little balls , while on one of these little balls moved a liliputian whom I should call myself. But this act of imagination would be impossible for me, had I not previously constructed my restricted space and my extended space for my own use.

IV.

Why have all these spaces three dimensions? Go back to the "table of distribution" of which we have spoken. We have on the one side the list of the different possible dangers; designate them by A1, A2, etc.; and on the other side, the list of the different remedies which I shall call in the same way B1, B2, etc. We have then connections between the contact studs or push buttons of the first list and those of the second, so that when, for instance, the announcer of danger A3 functions, it will put or may put in action the relay corresponding to the parry B4.

As I have spoken above of centripetal or centrifugal

wires, I fear lest some one see in all this, not a simple comparison, but a description of the nervous system. Such is not my thought for several reasons: first, I should not permit myself to put forth an opinion on the structure of the nervous system which I do not know, while those who have studied it speak only circumspectly; again, because, despite my incompetence, I well know this scheme would be too simplistic; and, finally, because on my list of parries, some would figure very complex, which might even, in the case of extended space, as we have seen above, consist of many steps followed by a movement of the arm. It is not a question then of physical connection between two real conductors, but of psychologic association between two series of sensations.

If A1 and A2 for instance are both associated with the parry B1, and if A1 is likewise associated with the parry B2, it will generally happen that A2 and B2 will also themselves be associated.

If this fundamental law were not generally true, there would exist only an immense confusion and there would be nothing resembling a conception of space or a geometry. How in fact have we defined a point of space? We have done it in two ways: it is on the one hand the aggregate of announcers A in connection with the same parry B; it is on the other hand the aggregate of parries B in connection with the same announcer A. If our law was not true, we should say A1 and A2 correspond to the same point since they are both in connection with B1; but we should likewise say they do not correspond to the same point, since A1 would be in connection with B2 and the same would not be true of A2. This would be a contradiction.

But, from another side, if the law were rigorously and always true, space would be very different from what it is. We should have categories strongly contrasted be-

tween which would be portioned out on the one hand the announcers A, on the other hand the parries B; these categories would be excessively numerous, but they would be entirely separated one from another. Space would be composed of points very numerous, but discrete; it would be *discontinuous*. There would be no reason for ranging these points in one order rather than another, nor consequently for attributing to space three dimensions.

But it is not so; permit me to resume for a moment the language of those who know geometry; this is quite proper since this is the language best understood by those I wish to make understand me.

When I desire to parry the stroke, I seek to reach the point from which the blow comes, but it suffices that I approach quite near. Then the parry B1 may answer for A1 and for A2, if the point which corresponds to B1 is sufficiently near both to that corresponding to A1 and to that corresponding to A2.

But it may happen that the point corresponding to another parry B2 may be sufficiently near the point corresponding to A1 and not sufficiently near the point corresponding to A2; so that the parry B2 may answer for A1 without answering for A2.

For one who is not acquainted with geometry, this translates itself simply by a derogation of the law stated above. And then things will happen thus: Two parries B1 and B2 will be associated with the same warning A1 and with a large number of warnings which we shall range in the same category as A1 and which we shall make correspond to the same point of space.

But we may find warnings A2 which will be associated with B2 without being associated with B1, and which in compensation will be associated with B3, which B3 was not associated with A1, and so forth, so that we may write the series

B1, A1, B2, A2, B3, A3, B4, A4,

where each term is associated with the following and the preceding, but not with the terms several places away. Needless to add that each of the terms of these series is not isolated but forms part of a very numerous category of other warnings or of other parries having the same connections, and which may be regarded as belonging to the same point of space.

The fundamental law, though admitting of exceptions, remains therefore almost always true. Only, in consequence of these exceptions, these categories, in place of being entirely separated, encroach partially one upon another and mutually penetrate in a certain measure, so that space becomes continuous.

On the other hand, the order in which these categories are to be ranged is no longer arbitrary, and if we refer to the preceding series we see it is necessary to put B2 between A1 and A2 and consequently between B1 and B3, and that we could not for instance put it between B3 and B4.

There is therefore an order, corresponding to the points of space, in which we naturally arrange our categories, and experience teaches us that this order presents itself under the form of a table of triple entry, and this is why space has three dimensions.

V.

So the characteristic property of space, that of having three dimensions, is only a property of our table of distribution, an internal property of the human intelligence, so to speak. It would suffice to destroy certain of these connections, that is to say of these associations of ideas, to give a different table of distribution, and that might be enough for space to acquire a fourth dimension.

Some persons will be astonished at such a result. The

external world, they will think, should count for something. If the number of dimensions comes from the way we are made, there might be thinking beings living in our world, but who might be made differently from us and who would believe space has more or less than three dimensions. Has not M. de Cyon said that the Japanese mice, having only two pair of semi-circular canals, believe that space is two-dimensional? And then would not this thinking being, if he is capable of constructing a physics, make a physics of two or of four dimensions, and which in a sense would still be the same as ours, since it would be the description of the same world in another language?

It seems in fact that it would be possible to translate our physics into the language of geometry of four dimensions; to attempt this translation would be to take great pains for little profit, and I shall confine myself to citing the mechanics of Hertz where we have something analogous. However it seems that the translation would always be less simple than the text, and that it would always have the air of a translation, that the language of three dimensions seems the better fitted to the description of our world, although this description can be rigorously made in another idiom.

Besides, our table of distribution was not made at random. There is connection between the warning A1 and the parry B1, this is an internal property of our intelligence; but why this connection? It is because the parry B1 affords means effectively to guard against the danger A1; this is a fact exterior to us, it is a property of the exterior world. Our table of distribution is therefore only the translation of an aggregate of exterior facts; if it has three dimensions, this is because it has adapted itself to a world having certain properties; and the chief of these properties is that there exist natural solids whose displacements follow sensibly the laws we call laws of motion of

rigid solids. If therefore the language of three dimensions is that which permits us most easily to describe our world, we should not be astonished; this language is copied from our table of distribution, and it is in order to be able to live in this world that this table has been established.

I have said we could conceive, living in our world, thinking beings whose table of distribution would be four-dimensional and who consequently would think in hyperspace. It is not certain however that such beings, admitting they were born there, could live there and defend themselves against the thousand dangers by which they would be assailed.

VI.

A few remarks to end with. There is a striking contrast between the roughness of this primitive geometry, reducible to what I call a table of distribution, and the infinite precision of the geometers' geometry. And yet the latter is born of the former, but not of that alone; it must be made fruitful by the faculty we have of constructing mathematical concepts, such as that of group, for instance. It was needful to seek among the pure concepts that which best adapts itself to this rough space whose genesis I have sought to explain and which is common to us and the higher animals.

The evidence for certain geometric postulates, we have said, is only our repugnance to renouncing very old habits. But these postulates are infinitely precise, while these habits have something about them essentially pliable. When we wish to think, we need postulates that are infinitely precise, since this is the only way to avoid contradiction; but among all the possible systems of postulates there are some we dislike to choose because they are not sufficiently in accord with our habits; however pliable, however elastic they may be, they have a limit of elasticity.

We see that if geometry is not an experimental science, it is a science born *à propos* of experience; that we have created the space it studies, but adapt it to the world wherein we live. We have selected the most convenient space, but experience has guided our choice. As this choice has been unconscious, we think it has been imposed upon us; some say experience imposes it, others that we are born with our space ready made. We see from the preceding considerations, what in these two opinions is the part of truth, what of error.

In this progressive education whose outcome has been the construction of space, it is very difficult to determine what is the part of the individual, what the part of the race. How far could one of us, transported from birth to an entirely different world, where were dominant, for instance, bodies moving in conformity to the laws of motion of non-Euclidean solids, renounce his ancestral space to build a space completely new?

The race seems indeed to play a preponderant part; yet if to it we owe rough space, the pliable space I have spoken of, the space of the higher animals, is it not to the unconscious experience of the individual we owe the infinitely precise space of the geometer?

This is a question not easy to solve. Yet we cite a fact showing that the space our ancestors have bequeathed us still retains a certain plasticity. Some hunters learn to shoot fish under water, though the image of the fish be displaced by refraction. Besides they do it instinctively. They therefore have learned to modify their old instinct of direction, or, if you choose, to substitute for the association $A1$, $B1$ another association $A1$, $B2$, because experience showed them the first would not work.

HENRI POINCARÉ.